



MATHEMATICAL OPTIMIZATION OF IRRIGATION PIPES

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INTRODUCTION

One of the biggest limitations of a pressurized irrigation system with great lengths is the high pipe installation cost. Mostly the pipes are fixed and if there is a need to extend the system to irrigate a larger area, the installation cost will keep on rising. To adjust the head loss in the longest pipes to the admitted loss, a composed pipes of some consecutive commercial diameters in the diverse stretches of nets of water distribution must be adopted. As a result, the designed pipe will have a lesser cost than that designed with only a single diameter.

OBJECTIVE

The objective of this work was, therefore, to develop a spread sheet based mathematical linear programming optimization tool that helps to select the optimum pipe size from four commercially available diameters under different conditions of contour. Minimizing the pipe cost was considered as an objective function.

METHODOLOGY

The pipe material considered was PVC, with a total length of 300 m. In order to study four different diameter combinations of pipes, the pipe was divided into two parts. The first part, which is at the entrance, had a length of 100 m; and the second part, which is at the end, had a length of 200m. The four commercially available internal diameters considered were 0.212m; 0.144m; 0.120m; 0.098m. The initial discharge amounts in the first and second parts of the pipe were 76.7 m³ h⁻¹ and 38.4 m³ h⁻¹, respectively. Following are the main hydraulic equations used in this study. The hydraulic head loss at the pipe entrance was calculated by:

$$H_L = h_L + k_1 h_f + 0.5 \Delta E l$$

where: H_L = hydraulic head loss at the entrance of the pipe (mH₂O); h_L = hydraulic load of operation demanded for the sender (mH₂O); k_1 = 0.75 for pipe with single diameter, 0.63 for pipes with two diameters and 0.5 for pipes with 3 or more diameters; h_f = head loss for attrition in the lateral line (mH₂O); $\Delta E l$ = elevation difference (m) between the beginning and the end of the pipe, being positive for pipes in uphill and negative for pipes downhill.

Equations of unitary head loss for attrition were used in the tests, with the objective to execute a comparative analysis of the values head loss, had been used different equations head loss for attrition for sizing of pipes (Hazen-Williams, Manning, Scobey, Swamee-Jain, Flamant and Darcy-Weisbach).

The head loss for attrition is calculated by the equation:

$$h_f = J_L JLF$$

Where: h_f - total head loss in the pipe (mH₂O); J_L - % of head loss (decimal); J - unitary head loss (m m⁻¹); L - length of the pipe (m); F - reduction factor.

The head loss in the reduction of the diameter is calculated by:

$$\Delta h_{f_{n2-fn1}} = K_n \frac{V_{n2}^2}{2g}$$

Where: $\Delta h_{f_{n2-fn1}}$ - head loss in the reduction of the diameter $n1$ for $n2$ (mH₂O); K_n - coefficient of head loss, according to relation of area of the pipe $n2$ com $n1$; V_{n2}^2 - speed of the water in the pipe $n2$ (m s⁻¹); g - acceleration of gravity (9.81 m (s²)⁻¹).

To find the coefficient of head loss, in the reduction of diameter of the tubing, an analysis of regression of the data supplied for hydraulic literature was executed:

$$K_n = 0.0817 (A2/A1)^6 + 0.5562 (A2/A1)^5 - 1.2032 (A2/A1)^4 + 1.0513 (A2/A1)^3 - 0.6362 (A2/A1)^2 - 0.3496 (A2/A1) + 0.5$$

Where: $A2/A1$ - relation enters the area of the pipe $n2$ and the pipe $n1$.

The objective function to be minimized was the total cost of the pipe, considering pipes of PVC with available 4 diameters different:

$$Min_{PRICE_{PVC}} = priceD1L1 + priceD2L2 + priceD3L3 + priceD4L4$$

Where: $Min_{PRICE_{PVC}}$ - total cost of the pipe, to be minimized, considering the add of the four stretches of different available diameters (US\$); $priceDnLn$ - cost of the n stretch of the pipe of PVC of diameter Dn and length Ln (US\$).

The variable were the lengths, with different diameters, of the four stretches of the pipe: $L1$; $L2$; $L3$; $L4$. The constraints of the model had been limitations of hydraulic pipe size of for nets of distribution of water and mathematical matrix, according to objective considered: $L_1 < L_2 < L_3 < L_4$, L_1 or L_2 or L_3 or $L_4 > 6$ m; L_1 and L_2 and L_3 and L_4 multiples of 6 meters; $L_1 + L_2 + L_3 + L_4 = 300$ m; < 4 mH₂O; $V1_{L1D1}$; $V2_{L2D2}$; $V3_{L3D3}$; $V4_{L4D4} < 2$ m s⁻¹.

RESULTS & DISCUSSION

Variations in length, discharge, and flow velocity that were considered and observed in this study are presented in Table 1.

The intrinsic characteristics of each equation of head loss, or either, each equation results in a different head loss which had to the inlaid variable. One can observe that the Hazen-Williams equation is the one that has a close total head loss with that of the universal head loss of Darcy-Weisbach, followed by the Manning and Scobey equation. The least pipe cost was found with the Scobey equation. However, it has the biggest total head loss. The flow velocity remained constant since the first derivation (one the 100 m and another one in the end of the pipe, the 300 m) was always in as the diameter of tubing, or either, already he was foreseen that in each stretch the speed was constant, therefore depends on the outflow and of the diameter used and these parameters they are constant in each stretch.

Table 1. Length (L_n) and speed (V_n) of each stretch, loss of total load (h_{total}) and total cost (in dollar), according to equation of head loss used in the sizing.

| Equation of head loss | L_1 | L_2 | L_3 | L_4 | V_1 | V_2 | V_3 | V_4 | h_{total} | Price |
|-----------------------|-------|-------|-------|-------|-------------------|-------|-------|-------|-------------------|-------|
| | m | | | | m s ⁻¹ | | | | mH ₂ O | US\$ |
| Hazen-Williams | 66 | 78 | 78 | 78 | 0.78 | 1.12 | 0.87 | 1.56 | 3.16 | 747.5 |
| Manning | 72 | 72 | 78 | 78 | 0.78 | 1.12 | 0.87 | 1.56 | 3.40 | 754.0 |
| Scobey | 6 | 98 | 98 | 98 | 0.78 | 1.12 | 0.87 | 1.56 | 3.88 | 610.6 |
| Swamee-Jain | 66 | 72 | 78 | 84 | 0.78 | 1.12 | 0.87 | 1.56 | 3.41 | 735.9 |
| Flamant | 54 | 60 | 78 | 108 | 0.78 | 1.12 | 0.87 | 1.56 | 3.87 | 675.8 |
| Darcy-Weisbach | 78 | 78 | 72 | 72 | 0.78 | 1.12 | 0.87 | 1.56 | 3.19 | 779.7 |

CONCLUSION

As result got the biggest possible lengths of pipe with the minors available commercial diameters. according to restrictions of hydraulic designed. Exactly having the diameters optimized under the usual hydraulic criteria. the choice of the equation of head loss in the sizing can intervene significantly with the costs of pipes.

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