Tropentag 2007
University of Kassel-Witzenhausen and
University of Göttingen, October 9-11, 2007

Conference on International Agricultural Research for Development

MATHEMATICAL OPTIMIZATION OF IRRIGATION PIPES

Francisco Marcuzzo[1]; Edson Wendland[1] & Lijalem Zeray Abraham[2]

[1] *University of São Paulo, Engineering School of São Carlos, Hydraulic Department, Brazil. (contact e-mail: fmarcuzzo@gmail.com)*
[2] *University of Applied Sciences Cologne, Institute for Technology in the Tropics (ITT), Germany*

Abstract

In order to secure increased quality on-farm food production, it is necessary to guarantee continuous water supply to the plants. However, this is difficult to be easily achieved due to the expensive overall irrigation systems` cost. Hence, high irrigation system installation cost is the biggest limitation, especially in the case of great length pipe systems. Consequently, the decision making process must consider different combinations of pipe lengths and pipe diameters for the design of irrigation systems. But this process is found to be very difficult for designers due to the fact that designers mostly made decision not on mathematically optimized way. Based on experience, the design engineer usually carries out countless trial-and-error computations to come up with a minimum acceptable head loss based on pre-established length and diameter combinations. Pipe design based on mathematical linear programming optimization is the best solution to solve this kind of problem. Accordingly, a mathematical linear programming based spreadsheet was developed for the design of the pipe system. The pipe considered has a length of 300 m combined with four different diameters of 212, 144, 120 and 98 mm; which are commercially available in the Brazilian pipe markets. The objective function that was minimized was the cost of the pipe; and the constraints considered were commercial availability of diameters, admissible head loss, and the highest flow velocity in the pipe. For the admissible head loss determination, six different head loss equations were tested. These are the Hazen-Williams, Manning, Scobey, Swamee-Jain, Flamant and Darcy-Weisbach equations. The lowest cost of US$ 610.59 was found by the Scobey head loss equation, whereas the highest cost of US$ 779.71 was observed by the Darcy-Weisbach head loss equation. The difference of 28% between the lowest and the highest costs indicates that the discrepancy that can be observed based on the decision made by the designers on selecting among the head loss equations.

Introduction

One of the biggest limitations of a pressurized irrigation system with great lengths is the high pipe installation cost. Mostly the pipes are fixed and if there is a need to extend the system to irrigate a larger area, the installation cost will keep on rising. SAAD *et al.* (1994) indicated that, targeting a lesser installation cost, the fact of fixed pipe irrigation system makes use of diameters combinations involving distinct and different optimized lengths possible. To adjust the head loss in the longest pipes to the admitted loss, a composed pipe of some consecutive commercial diameters in the diverse stretches of nets of water distribution must be adopted. As a result, the designed pipe will have a lesser cost than that designed with only a single diameter (GOMES, 1999).

According to LISBON (2002), the formulation of the problem to be optimized follows some basic steps: 1) either the basic objective of the problem or the optimization to be achieved (objective function) must be defined; 2) this objective function must be defined based on the mathematically specified decision variables involved; 3) these variable are normally parts of a series of constraints normally represented with equations. SAAD & MARCUSSI (2006) reported that the linear mathematical optimization is an excellent tool for the operational research in hydraulical parameters of pipes. Since the contour conditions are normally lines, thus making possible excellent a global one enters the different possibilities of sizing.

The objective of this work was, therefore, to develop a spread sheet based mathematical linear programming optimization tool that helps to select the optimum pipe size from four commercially available diameters under different conditions of contour. Minimizing the pipe cost was considered as an objective function.

Material and Methods

The pipe material considered was PVC, with a total length of 300 m. In order to study four different diameter combinations of pipes, the pipe was divided into two parts. The first part, which is at the entrance, had a length of 100 m; and the second part, which is at the end, had a length of 200m. The four commercially available internal diameters considered were 0.212m; 0.144m; 0.120m; 0.098m. The initial discharge amounts in the first and second parts of the pipe were 76.7 $m^3\ h^{-1}$ and 38.4 $m^3\ h^{-1}$, respectively.

Following are the main hydraulic equations used in this study. The hydraulic head loss at the pipe entrance was calculated by (KELLER & BLIESNER, 2000):

$$H_L = h_L + k_1\, h_{f_1} + 0{,}5\Delta El \tag{1}$$

where: H_L = hydraulic head loss at the entrance of the pipe (mH_2O[1]); h_L = hydraulical load of operation demanded for the sender (mH_2O); k_1 = 0.75 for pipe with single diameter, 0.63 for pipes with two diameters and 0.5 for pipes with 3 or more diameters (KELLER & KARMELI, 1974); h_{f1} = head loss for attrition in the lateral line (mH_2O); ΔEl = elevation difference (m) between the beginning and the end of the pipe, being positive for pipes in uphill and negative for pipes downhill.

Equations of unitary head loss for attrition were used in the tests, with the objective to execute a comparative analysis of the values head loss, had been used different equations head loss for attrition for sizing of pipes (Hazen-Williams, Manning, Scobey, Swamee-Jain, Flamant and Darcy-Weisbach - Source: PORTO, 1998; AZEVEDO NETTO *et al.*, 1998; GOMES, 1999).

The head loss for attrition is calculated by the equation (GOMES, 1999):

$$h_f = J_L JLF \tag{2}$$

Where: h_f - total head loss in the pipe (mH^2O); J_L - % of head loss (decimal); J - unitary head loss ($m\ m^{-1}$); L - length of the pipe (m); F - reduction factor.

[1] Water column

The head loss in the reduction of the diameter is calculated by (PORTO, 1998):

$$\Delta h_{fn2-fn1} = K_n \frac{V_{n2}^{\ 2}}{2g} \qquad (3)$$

Where: $\Delta h_{fn2-fn1}$ - head loss in the reduction of the diameter *n1* for *n2* (mH_2O); K_n - coefficient of head loss, according to relation of area of the pipe *n2* com *n1*; $V_{n2}^{\ 2}$ - speed of the water in the pipe *n2* (m s^{-1}); g - acceleration of gravity (9.81 m $(s^2)^{-1}$).

To find the coefficient of head loss, in the reduction of diameter of the pipe, an analysis of regression of the data supplied for PORTO (1998) was executed.

$$K_n = 0.0817(A2/A1)^6 + 0.5562(A2/A1)^5 - 1.2032(A2/A1)^4 + 1.0513(A2/A1)^3 - 0.6362(A2/A1)^2 - 0.3496(A2/A1) + 0.5 \qquad (4)$$

Where: $A2/A1$ - relation enters the area of the pipe *n2* and the pipe *n1*.

The objective function to be minimized was the total cost of the pipe, considering pipes of PVC with available 4 diameters different:

$$Min_{f(PRICEPVC)} = priceD1L1_{PVC} + priceD2L2_{PVC} + priceD3L3_{PVC} + priceD4L4_{PVC} \qquad (5)$$

Where: $Min_{f(PRICEPVC)}$ - total cost of the pipe, to be minimized, considering the add of the four stretches of different available diameters (US$); $priceDnLn_{PVC}$ - cost of the *n* stretch of the pipe of PVC of diameter *Dn* and length *Ln* (US$).

The variable for this model of linear otimisation were the lengths, with different diameters, of the four stretches of the pipe: *L1; L2; L3; L4.*

The constraints of the model had been limitations of hydraulical pipe size of for nets of distribution of water and mathematical matrix, according to objective considered: $L_1<L_2<L_3<L_4$; L_1 or L_2 or L_3 or L_4 > 6 meters (condition so that the program uses the four pipes of available different diameters); L_1 and L_2 and L_3 and L_4 multiples of 6 meters (commercial condition of the PVC pipeline); $L_1 + L_2 + L_3 + L_4 = 300$ meters; h_f < 4 mH_2O; $V1_{L1D1}$; $V2_{L2D2}$; $V3_{L3D3}$; $V4_{L4D4}$ < 2 m s^{-1}.

Results and Discussion

Variations in length, discharge, and flow velocity that were considered and observed in this study are presented in Table 1.

The intrinsic characteristics of each equation of head loss, or either, each equation results in a different head loss which had to the inlaid variable. One can observe that the Hazen-Williams equation is the one that has a close total head loss with that of the universal head loss of Darcy-Weisbach, followed by the Manning and Scobey equation.

The least pipe cost was found with the Scobey equation. However, it has the biggest total head loss. The flow velocity remained constant since the first derivation (one the 100 m and another one in the end of the pipe, the 300 m) was always in as the diameter of pipe, or either, already he was foreseen that in each stretch the speed was constant, therefore depends on the outflow and of the diameter used and these parameters they are constant in each stretch.

Table 1. Length (L_n) and speed (V_n) of each stretch, loss of total load (h_{ftotal}) and total cost (in dollar), according to equation of head loss used in the sizing.

Equation of head loss	L_1	L_2	L_3	L_4	V_1	V_2	V_3	V_4	h_{ftotal}	Price
	m				m s^{-1}				mH_2O	US$
Hazen-Williams	66	78	78	78	0.78	1.12	0.87	1.56	3.16	747.51
Manning	72	72	78	78	0.78	1.12	0.87	1.56	3.40	754.04
Scobey	6	98	98	98	0.78	1.12	0.87	1.56	3.88	610.59
Swamee-Jain	66	72	78	84	0.78	1.12	0.87	1.56	3.41	735.38
Flamant	54	60	78	108	0.78	1.12	0.87	1.56	3.87	675.84
Darcy-Weisbach	78	78	72	72	0.78	1.12	0.87	1.56	3.19	779.71

Conclusions

As result got the biggest possible lengths of pipe with the minors available commercial diameters. according to restrictions of hydraulical designed. Exactly having the diameters optimized under the usual hydraulical criteria. the choice of the equation of head loss in the sizing can intervene significantly with the costs of pipes.

References

AZEVEDO NETTO. J.M.; FERNANDEZ Y FERNANDEZ. M.; ARAUJO. R. DE; ITO. A.E. **Manual de hidráulica**. 8. ed. São Paulo: Blücher. 1998. 669p.

GOMES. H.P. **Engenharia de irrigação**. 3. ed. Campina Grande: Universidade Federal da Paraíba. 1999. 412 p. AZEVEDO NETTO. J.M.; FERNANDEZ Y FERNANDEZ. M.; ARAUJO. R. DE; ITO. A.E. **Manual de hidráulica**. 8. ed. São Paulo: Blücher. 1998. 669p.

KELLER. J.; BLIESNER. R. D. **Sprinkle and trickle irrigation**. New Jersey: Caldwell. The Blackburn Press. v. 1. 652 p. 2000.

KELLER. J.; KARMELI. D. Trickle irrigation design parameters. **Transaction of the ASAE.** v. 17. p. 679-684. 1974.

LISBOA. E. F. A. **Pesquisa operacional.** Apostilado curso de pesquisa operacional. Rio de Janeiro. RJ: versão digital disponível na internet: http://www.ericolisboa.eng.br. 2002. 56 p.

PORTO. R.M. **Hidráulica básica**. São Carlos: EESC-USP. 1998. 540p.

SAAD. J.C.C.; MARCUSSI. F.F.N. **Distribuição da carga hidráulica em linhas de derivação otimizadas por programação linear**. Revista Engenharia Agrícola. Jaboticabal. v. 26. n.2. p. 406-414. maio/ago. 2006.

SAAD. J.C.C.; TOMAZELA. C.; PERES. J.G.; PERES. F.C.; FRIZZONE. J.A. Otimisation de rede hidráulica de um sistema de irrigação por gotejamento utilizando programação linear. **Revista Pesquisa Agropecuária Brasileira**. v.29. n.5. p. 797-805. 1994.