Deutscher Tropentag 2003
Göttingen, October 8-10, 2003
Conference on International Agricultural Research for Development

Results from a spatial water allocation model (SWAM) for water efficiency and irrigation technology choices: A case study from Northwestern China

Lan Fang and E.-A. Nuppenau*

* Universität Giessen, Institut für Agrarpolitik und Marktforschung, Scenkenbergstr. 3, 35390 Giessen, Germany,
Email: Lan.Fang@agrar.uni-giessen.de

Abstract
The issue of efficient water use has attracted much attention. For instance, China is facing severe water shortage due to geographical and demographic arguments. Especially under-developed water conservation technology and inefficient management are big problems for farmers and the government. Major questions at hand are how to improve water use efficiency, to optimize water allocation in agriculture, to invest in water saving technologies, and to assure water for high value added agriculture. This paper investigates the impact on water use efficiency by taking into account individual farmers' adoption of modern water saving technologies and improvements of water transit, contributed by the public sector, from sources to end of canals. It shows results from a spatial water allocation model (SWAM) according to the approach of Umetsu. The main contribution of the study is to optimize the water allocation and choices of irrigation technology for farmers in a water project.

Key words: Spatial model; Water efficiency; Public investment; Private investment

1 Introduction

In order to deal with the water scarcity problem, public and private sectors have made tremendous efforts worldwide. The aim of this paper is to provide policy makers with a theoretical and quantitative tool to manage public water supply and conveyance system more efficiently and to support the optimal allocation of water for irrigation projects. Furthermore, the current situation and potential likelihood of adopting modern irrigation technology are taken into account for private individual.

To achieve the objectives of this study, a spatial mathematic model, SWAM, was designed to assess the impacts of public and private investment on social welfare and water resource allocation. By analyzing the different status of public and private investment in an irrigation project area, this study seeks specifically (1) To determine the optimum amounts of surface and groundwater consumption at different locations in an irrigation project. (2) To investigate the efficiency of water conveyance systems supported by public investment. (3) To investigate on-farm water use efficiency by analyzing the private investment undertaken. (4) To explore the relationship between public and private investment. (5) To analyze different impacts on social economy and water resource allocation by considering different amounts of public and private investment.

2 Methodology

The methodological objective of the study was to build a comprehensive modeling framework. The framework contains two packages. One is an econometric model using regression methods (SPSS). The other is a mathematical programming model employing the General Algebraic Modeling System (GAMS). The empirical data for the study was collected from a northwestern Chinese county, Li Quan, Shaanxi Province in 2000, in which the farming system is dominated by apple production. Farmers ensure their food security by growing apples for food exchange. The natural conditions of agricultural activities, such as soil quality, climate, etc., are assumed constant and excluded. Whereas the heterogeneity of location along the public canal was given priority in the optimization. The model's usefulness is, therefore, not regionally confined.

To deal with the econometric model, Hotelling’s Lemma is applied to obtain a net revenue function by integrating an inverse water demand function. Furthermore, the on-farm water use efficiency function, the water loss function, and the functions for the canal water price and the groundwater price have been estimated by using regression methods. These functions served as key components in the dynamic mathematical model.

To deal with the programming model, according to the approach of Umetsu (Umetsu & Chakravorty, 1998), a spatial model has been established (Nuppenau & Fang, 2001). The spatial model incorporated an objective function and several constraints. The objective function was used to maximize the social welfare (producer surplus) in the survey area by focusing on efficient use of water. The optimization of social welfare was investigated by considering the water related net revenue of the survey area minus the expenditure on water conservation and other water costs.

The constraints included the on-farm water use efficiency function, the water loss function and equations of motion on water movement. In particular, the equations of motion are the most important constraints in the spatial model (Chiang, 1992; Dellink, Szonyi, & Bartelings, 2001; McKinney & Savitsky, 2003). Due to the high non-linear characteristics of the objective function and constraints, the model was solved by using Conopt (GAMS solver) and Minos (GAMS solver), together.

2.1 Specification of objective function

The objective function of the spatial model is calibrated with the estimated profit function. The programming model considers only water-related profits, whereas non-water related profits are not concerned in the study.

The objective function of the model is specified below:
Social welfare = Net revenue – Private investment in irrigation technology – Public investment in the water conveyance system – Canal water costs – Groundwater costs (1)

In line with a mathematical formulation the objective function can be presented as below:

$$Max\ SW = 15\left(\sum_j \pi_j - \sum_j I_j - \sum_j CWP_j \times CW_j - \sum_j GWP_j \times CW_j\right) - 0.05\sum_j K_j \qquad (2)$$

Where j = location, it ranges from 1 to 200, and it represents a stretch every 50m along the canal, i.e., the system stretch is 10km.
sw= social welfare over the irrigation area
π_j = Net revenue in Yuan/ Mu at location j
I_j = private investment in technology in Yuan/Mu at location j
K_j = public investment in water conveyance in Yuan/Kilometer at location j
CWP_j = price of canal water at location j
GWP_j = price of groundwater at location j
CW_j = canal water consumption at location j
GW_j = groundwater consumption at location j

Employment of coefficient "15" is Mu related. It converts to Chinese land measurement in hectare.

The first argument of the right hand side (RHS) $15\sum_j \pi_j$ represents the sum of net revenue of every unit at location j. A stretch of j is 50 meters long; the length of the whole project area is assumed 10,000 meters long and 200 meters wide. It suggests that one unit area what j represents is 50×200 = 10,000m^2, which is either equal to 1 hectare or 15 Mu. Since π represents the net revenue per Mu, $15\sum_j \pi_j$ is therefore a representation of the sum of net revenues over all locations. The second argument $15\sum_j I_j$ of the RHS represents the sum of private investment in irrigation technology at every location *j*. The third and fourth two arguments in the bracket are canal water costs and groundwater costs respectively. The last argument $0.05\sum_j K_j$ of the RHS represents the sum of public investment in the canal system at every location *j*. Since K is measured in Yuan/kilometer, one unit of *j* (50m) is equivalent of 0.05 length of one kilometer.

2.2 Specification of components of the objective function and constraints of the model

The net revenue function and the on-farm water use efficiency function are the key components of the objective function of the programming model. Based on empirical data, they are presented follows:

The net revenue function:

$$\pi_j = 11.83EW_j - 0.01EW_j \times I_j - 0.07EW_j^{\ 2} \qquad (3)$$

Where EW_j is the effective water use, which will be replaced by $TW_j \times h_j$ in the programming model, I_j is the investment. So the net revenue function is newly specified as:

$$\pi_j = 11.83 \times (TW_j \times h_j) - 0.01 \times (TW_j \times h_j) \times I_j - 0.07 \times (TW_j \times h_j)^2 \qquad (4)$$

Where TW_j is the total water demand in the area of 1Mu and h_j is the on-farm water use efficiency at location *j*

The on-farm water use efficiency function: is

$$h_j = 0.48 + 0.0025 \times I_j - 2.94 \times 10^{-6} \times I_j^{\ 2} \qquad (5)$$

The water use efficiency function is directly used in the objective function as one component of the net revenue function. It contributes to the measurement of the effective water consumption, but also serves as one constraint.

The water loss rate function is:

$$a_j = 0.74 - 0.000405 \times K_j + 5.25 \times 10^{-7} \times K_j^{\ 2} \qquad (6)$$

Where a_j is the water loss rate at location *j*, K_j is annual public investment per kilometer at location *j*. This water loss function will serve as a component of the equations of motion for canal water and groundwater and as one constraint of the optimization process as well.

Equations of motion are the most important constraints in a dynamic model. In this model they are transferred as location wise function. Technically one speaks of differential equations. For us, they are the central elements to solve a spatial location problem. Since canal water is moving between locations and the groundwater stock also changes at locations, differential equations can be expressed as equation of spatial motion respectively.

The equation of motion for canal water flow in this model is thus expressed as a way of requirements of GAMS (McKinney & Savitsky, 2003; Dellink, Szonyi, & Bartelings, 2001). It can be specified as below:

Initial condition:

$$crem_1 = cw_0 - 15 \times cw_1 \tag{7}$$

Continuous flows (discrete):

$$crem_j = (1 - a_{j-1}) \times crem_{j-1} - 15 \times cw_j \tag{8}$$

The equation (7) is the initial condition for canal water flow, where cw_0 represents the canal water supply at the water source. "cw_1" is the quantity of canal water consumed by the first farmer within first 50 meters, $crem_1$ is therefore the canal water that remains after the first farmer and then passes down to the next farmer, i.e., the next location.

Then equation (8) describes the amount of canal water that remains at location j, which starts from second farmer and is going to be delivered to the next farmer. It is the general function of motions. It is expressed as a value of water that remains from the previous location j-1 minus water consumption at the present location j. Where $crem_j$ represents the canal water that remains, i.e., the canal water stock at location j. In the equations, $crem_{j-1}$ represents the canal water that remained from the previous farmer's location j-1. Additionally we introduce the water loss rate a_{j-1} defined as before, it represents the water loss rate at location j-1.

This was the canal water movement. In principle, the equation of motion is the same for groundwater motion. The initial point starts from the head of the survey area. Specifically "$grem_1$" is the groundwater remaining at the first location of the survey area. In terms of terminal condition, however, groundwater is free of restrictions and gives a lower bound of zero in the optimization process. The only difference is, that the groundwater extraction starts at the point where canal water is used up instead from the first location (water source) as for canal water.It is important to recognize that the groundwater aquifer can be recharged by water leaking from the canal and seepage from farmer's fields. It is therefore, suggested that the groundwater stock will increase all the time due to the recharge from both sources and without any extraction before farmers start to take groundwater. It is further noticed that, the canal water is so cheap that farmers have no incentive and no need to pump underground water until canal water is used up. After farmers switch to groundwater, the fraction recharged from canal water becomes zero. The groundwater stock can only be recharged by seepage from farmer's fields. These stages will be also specified in the equation of motion. The mathematical formulation of the equation of motion for groundwater change is presented as below:

Initial condition:

$$grem_1 = gw_0 + \beta \times (1 - h_1) \times 15 \times tw_1 - 15 \times gw_1 \tag{9}$$

Continuous motion (discrete):

$$grem_j = grem_{j-1} + \beta \times a_{j-1} \times crem_{j-1} - 15 \times gw_j + \beta \times (1 - h_j) \times 15 \times tw_j \tag{10}$$

Equation (9) describes the initial condition for groundwater change. Where $grem_1$ represents the groundwater remaining at location 1 which will be available for the second location, gw_0 represents the groundwater base stock at the head location, i.e., the groundwater stock at the first farmer's field. The second part is the fraction of groundwater recharged from the first farmer's field, since no water recharged from canal is observed at the first location due to zero distance from the water source. In this part, tw_1 is the joint conjunctive water used at the first location, β is defined as the recharge rate for groundwater, and h is the water efficiency in farmers' field. gw_1 is the groundwater consumption at the first location. Equation (10) is the change of groundwater stock at any location except the first location. The $grem_j$ represents the groundwater remaining from the previous the farmer to the next farmer at location j, here j starts from farmer 2. β is recharge rate for groundwater, so $\beta \times a_{j-1} \times crem_{j-1}$ represents the water loss fraction at the location j-1 from the canal and can be recharged to the aquifer. gw_j is the groundwater quantity extracted by an individual farmer at location j. As defined before, h is the effective water use function, therefore $\beta \times (1 - h_j) \times 15 \times tw_j$ represents the fraction of pumped groundwater and surface water loss from a farmers' field which recharges the groundwater aquifer at location j. It can be used by farmers at the next location

These equations of motion for canal water flow and groundwater change for each location will serve as the most important constraint conditions in this spatial programming model.

3 Scenarios design and simulation results

Three scenarios were designed in order to evaluate different impacts of public investment status change on social welfare and water resources allocation. In particular, private investment, public investment as well as the relationship between them are the main concerns of the study.

The base scenario (LSEK) was designed based on endogenous public and private investment and the latter two scenarios LSRK and HSRK were tested by removing public investment from the water conveyance system exogenously. Table 1 and 2 analyze the different impacts of public investment change social economy and water resource allocation at aggregate and farm level respectively.

In Table 1, it is surprisingly observed that the aggregate social welfare achieved in scenario HSRK is 1,062,253.93 Yuan, which is only 0.29% lower than in scenario LSEK, and 73.35% higher than in scenario LSRK. It indicates, because of higher soil permeability and recharge rate,

Table 1: Comparison of impacts on social welfare and water resource allocation at aggregate level among scenario LSEK, LSRK and HSRK

Items	HSRK	LSRK	LSEK	%(LSRK& LSEK)	%(HSRK& LSRK)	%(HSRK& LSEK)
Social welfare (Yuan)	1062253.93	612782.89	1065334.88	-42.48	73.35	-0.29
Total canal water consumption (m^3)	76542.66	66027.23	300000.00	-77.99	15.93	-74.49
Total groundwater consumption (m^3)	362341.81	96554.60	56635.07	70.49	275.27	539.78
Total water consumption (m^3)	438884.47	162581.82	356635.07	-54.41	169.95	23.06
Capacity of water supply (m^3)	301000.00	301000.00	301000.00	0.00	0.00	0.00
Gain from conjunctive water use (m^3)	137884.47	-138418.18	55635.07	-348.80	199.61	147.84
Total public investment (Yuan/km)	0.00	0.00	2431.55		0.00	
Switch point (Location)	31.00	37.00	164.00	-77.44	-16.22	-81.10
Canal water length (m)	1550.00	1850.00	8200.00	-77.44	-16.22	-81.10
Area irrigated by canal water (Mu)	465.00	555.00	2460.00	-77.44	-16.22	-81.10
Area irrigated by groundwater (Mu)	2535.00	2445.00	540.00	352.78	3.68	369.44
Total private investment (Yuan)	0.00	0.00	0.00	0.00	0.00	0.00

Notes: the switch point of scenario HSRK is actually observed at location 32, on which canal water ends and groundwater starts. Complementary water is used at this point. *cw*32=10.36m3, *gw*32= 144.27m3. The consumption of canal water is very small compared to that of groundwater at location 32, so that the canal water consumption at location 32 is neglected while calculating the canal water length and area irrigated by canal water. It therefore suggests a switch point at location 31 instead of 32.

HSRK: A removal of public investment under high soil permeability, endogenous private investment.

LSRK: A removal of public investment under low soil permeability, endogenous private investment

LSEK: Base scenario, low soil permeability, endogenous public and private investment that groundwater use becomes more available and profitable. Due to the considerable canal water loss rate (without lining investment) and high recharge rate, the total canal water consumption slightly increases by 15.93% compared to scenario LSRK, but dramatically decreases by 74.49% compared to scenario LSEK. Remarkable change appears in groundwater consumption. Total groundwater consumption reaches 362,341.81m^3, 2.75 times higher than in scenario LSRK, and 5.4 times higher than that in scenario LSEK. Consequently the total water consumption over the entire irrigation area is 4,388,824.47m^3, which is 1.7 times higher than in scenario LSRK and 23.06% more than it in scenario LSEK. In the current scenario, the gain from conjunctive water use is quite significant. It achieves 137,884.47 m^3, almost half of the capacity of water supply. It is correspondingly 1.48 times of the gain in scenario LSEK. These results seem not correct intuitively. The reason for these results is, that the higher recharge rate results in groundwater being able to be pumped more conveniently and abundantly. Due to high soil permeability, farmers have to pump more frequently to meet their water requirement. The model calculates these volumes of accumulated pumping and re-pumping, it hence results in such a bigger total water consumption, even much more than its actual capacity. In other words, water is re-used in the current scenario, and it is calculated as long as it is taken place. It is noticed that the area irrigated by canal water decreases sharply due to huge water losses from the canal system and farmer's fields. The canal water is almost used up already at location 31, i.e., after 1550 meters, the shortest distance within the three scenarios. On the one hand, the irrigated area by canal water is down to 465 Mu, a decrease by 16.22% and 81.1% respectively compared to scenario LSRK and scenario LSEK. On the other hand, the area irrigated by groundwater grows dramatically, with a considerable increase by 3.68% and 369.44% respectively compared to scenario LSRK and scenario LSEK.

Table 2 gives more detailed information about the impacts on indicators at farm level in the irrigation area. The average unit water consumption is reaching up to 146.3m^3/Mu for AU (All water users) in scenario HSRK, which is the highest water consumption level among the three scenarios, mainly due to re-pumping water from underground. It is 2.7 times higher than that in scenario LSRK, and 23.06% higher than that in scenario LSEK. In terms of canal water use, the mode result suggests the same up-tendency. But the consumption volume increases not so significantly as compared to that for AU. On average the unit canal water consumption is 164.61m^3, ranking in the top place among the three scenarios. In terms of groundwater consumption, a considerable difference between the current scenarios HSRK and the previous two is observed. Unit groundwater consumption for GWU (Groundwater users) in scenario HSRK also ranks in the highest position among the three scenarios. It reaches a volume of 142.94 m^3, 3.7 times that in scenario LSRK and 36.29% more than in scenario LSEK. High seepage rates result in less water availability in canal and huge groundwater recharge, and hence enforce farmers to pump groundwater. Variables, such as revenue, land rent, and water rent are following

Table 2: Comparison of mean value of indicators among scenario LSEK, LSRK and HSRK at farm level

	AU			CWU			GWU		
Items	HSRK	LSRK	LSEK	HSRK	LSRK	LSEK	HSRK	LSRK	LSEK
Aggregate water demand (m^3)	146.30	54.19	118.88	164.61	118.97	121.95	142.94	39.49	104.88
Revenue (Yuan)	484.08	242.52	445.26	497.30	436.17	451.24	481.62	198.56	418.02
Land rent (Yuan)	354.05	204.26	355.92	457.78	406.75	371.73	335.05	158.30	283.93
Water rent (Yuan)	129.96	38.25	88.53	39.51	29.42	78.53	146.55	40.26	134.09
Private investment (Yuan)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
On-farm water use efficiency	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
Public investment (Yuan)	0.00	0.00	243.16	0.00	0.00	298.35	0.00	0.00	0.00
Conveyance water loss rate	0.07	0.07		0.07	0.07	0.00	0.07	0.07	0.07

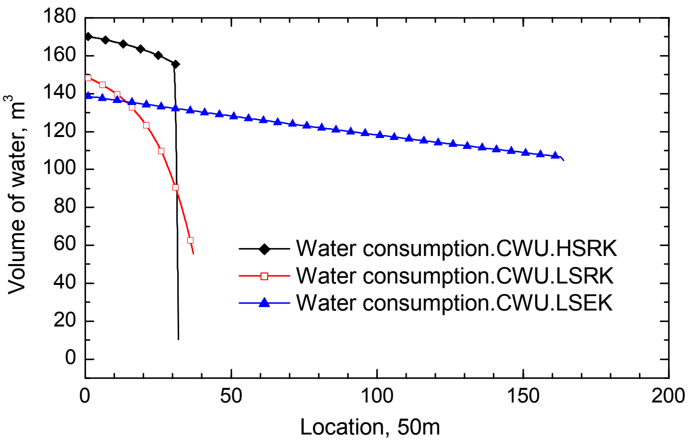
Notes: Mean analysis is made based on the number of CWU being 31 and GWU being 169.AU: All users; CWU: canal water users;

GWU: groundwater users;

HSRK: A removal of public investment under high soil permeability, endogenous private investment.
LSRK: A removal of public investment under low soil permeability, endogenous private investment
LSEK: Base scenario, low soil permeability, endogenous public and private investment

the tracks of movement of joint water consumption. They show a strong signal of overall better off in scenario HSRK.

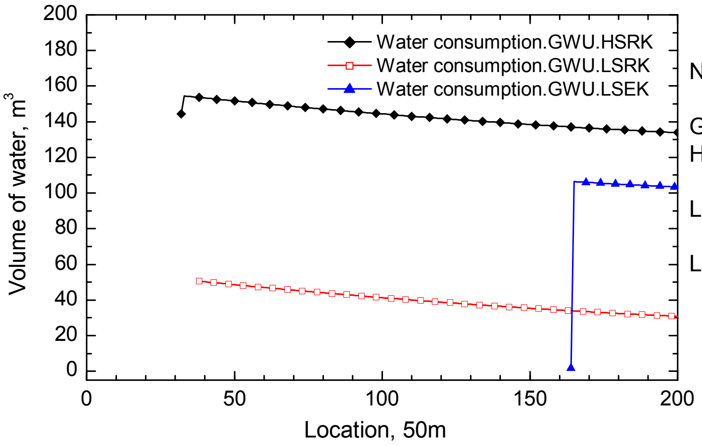
A comparison of the change in canal water consumption over distance among scenario HSRK, scenario LSRK and scenario LSEK is illustrated in Figure 1. In scenario HSRK, the canal water starts from the top level but ends quickly over a short distance. In scenario LSRK it also starts from a relatively high level and is used up quickly too. The common reason for quick ending of canal water use is the removal of public investment in both scenarios, so that the water in the canal suffers a high base water conveyance loss rate of 0.07. In scenario LSEK the canal water has traveled the longest distance, due to its improved water conveyance system.



Notes:

CWU: Canal water users
HSRK: A removal of public investment under high soil permeabilty
LSRK: A removal of public investment under low soil permeabilty
LSEK: Base scenario of endogenous public investment under low soil permeabilty

Figure 1: Comparison of canal water consumption at different locations among scenario HSRK, LSRK and LSEK



Notes:

GWU: Groundwater users
HSRK: A removal of public investment under high soil permeabilty
LSRK: A removal of public investment under low soil permeabilty
LSEK: Base scenario of endogenous public investment under low soil permeabilty

Figure 2:Comparison of groundwater consumption at different locations among scenario HSRK, LSRK and LSEK

In the following Figure 2, we can compare the unit groundwater consumption under the three scenarios. It is easy to understand that the track of groundwater movement is exactly opposite to that of canal water. For instance, as shown in Figure 1, canal water in scenario HSRK travels the shortest distance; hence in Figure 2, groundwater use, in scenario HSRK, appears early at location 32 and travels the longest distance.

Both figures indicate, that the tendency of canal water movement changes more dramatically than that of groundwater movement. The reason for this is, that canal water movement is influenced by a double loss of water, i.e., conveyance water loss and on-farm water loss, and moreover the farmers' extraction. Canal water therefore, is reduced faster than groundwater. Groundwater movement can be influenced by the on-farm water loss rate and less extraction by groundwater users. Consequently they perform in such a tendency.

4 Discussions and conclusions

The three scenarios have focused on investigating the impact of changing public investment status. The model results suggested optimal solutions for an irrigation system with moderate and high soil permeability, respectively.

Since the private investment in irrigation technology is a very heavy expenditure for farmers, scenario LSEK internally determined a zero investment in irrigation technology over all locations in the model. Based on this result, the further selected scenarios hence value the impacts on social welfare and water resource allocation merely by focusing on the role the public investment plays. They have been undertaken with public investment being made or removed under two different soil conditions. If the soil permeability is modest, public investment will largely improve the aggregate social welfare and water resource allocation. However if the soil permeability is very high, an irrigation system without public investment shows that the social welfare and water resource allocation are only slightly worse off as compared to scenario LSEK. Farmers are much better off as compared to scenario LSRK, which is with a low recharge rate. This indicates clearly, that a suitable policy or public expenditure with respect to a canal system is needed. It has to take the local natural condition, such as climate, soil condition etc, into account. As discussed already, the status of soil permeability is so important that it can have totally different impacts on the same project. In an area with low soil permeability, it is necessary to invest more in the water conveyance system, as shown in scenario LSEK as compared in an area of high permeability. However, if an area has very high soil permeability, and, if the local community is facing budget shortage, it might be wise not to invest much into the canal system as demonstrated in scenario HSRK. The model results of scenario HSRK actually suggest a basin wide optimal rather than a point optimal solution.

The performances of private investment can be modeled either by increasing the output price level or employing an additional coefficient. This part of work is not capable to present in this paper due to limitation of space.

References

CHIANG, A.C. (1992) *Elements of Dynamic Optimization*. McGraw-Hill.

DELLINK,R., SZONYI,J. & BARTELINGS,H. GAMS-for environmental -economic modeling. Environmental Economics and Natural Resources Group, University of Wageningen, the Netherlands. 2001.
Ref Type: Electronic Citation

MCKINNEY, D.C. & SAVITSKY, A.G. Basic Optimization Models For Water And Energy management. www.gams.com. 2003.
Ref Type: Electronic Citation

NUPPENAU,E.-A. & FANG,L. Water efficiency and choices on technology choices for irrigation system: a case study on farmers' behavior in the Chinese Province Shannxi. Deutscher Tropentag 2001, Bonn. 2001.
Ref Type: Electronic Citation

UMETSU,C. & CHAKRAVORTY,U. (1998) Water conveyance, return flows and technology choice. *Agricultural economics: the journal of the international association of agricultural economics* **19,** 181-191.